Category Theory

Category, Category Object, Maps And Composition, Functor, Natural Transformation, Adjoint

Category theory is a foundational area of mathematics that examines in a uniform manner mathematical structures and their mappings. In category theory, a category is a mathematical universe. A category is populated by category objects and there are mappings (also called morphisms) between these objects.

A category is itself a mathematical object. There can be mappings between categories, known as functors, and even mappings between such functors, known as natural transformations. Categories are abstract representations of concepts from other areas of mathematics.

It is best to think of the goal of category theory is to describe an abstract multiverse, containing one or more universes, with mappings within each universe, and between them. From this surprisingly simple core of constructs, a rich description of much of modern mathematics can accurately be built.

This course helps mathematicians and software developers gain an appreciation of what is category theory, both basic concepts and more advanced capabilities, and to see see how it can be practically applied in real-world situations.

Target Audience

This course is aimed at mathematicians and software engineers interested in learning about category theory

Prerequisites

Good foundational mathematical education along with some programming experience, as we include exploring category theory from a computational viewpoint. Attendees can select which programming language they wish to use, as all concepts will be developed from first principles.

Contents of One-Day Training Course

Category Theory Overview

Why the interest?
What is a category?
Practical applications
Examining structure-preserving mappings between objects

Basic Ideas

Introduction to the basic constructs
A category object is a mathematical object (to begin with, think of it as a set of elements) - so is not the same as an object in typical OO programming
Think of a mapping (morphism) between category objects as a relation
Maps can be composed – so if we have map f and map h, we compose them as h o f (read as “h follows g”)
Such compositions form paths

Mathematical Definition

A category is defined as a collection of category objects, with maps (with domain and codomain) between them
Each object has an identity map
Two important laws:
* Identity Law governs identity map use
* Associative Law: (h o g) o f = h o (g o f)

Functors

Functors are mappings between categories
Structure-preserving
Endofunctors are mappings from a category object to itself -”endo” (as in “endoscope”) means look inside oneself
Left/right adjoint functors (as in adjoining)

Exploring Sample Categories

Set, Ring, Monoidal, Group, 2-category, custom categories

Advanced Category Theory

Natural transformations
A deeper look at a variety of morphisms
Limits and colimits
Universal property
Initial and terminal object
Presheaf

∞-groupoid

“Sets in the next dimension are groupoids”
Groupoid builds on group in group theory
A groupoid is a category where each morphism is an isomorphism
∞-groupoid generalizes groupoids
k-morphisms and equivalences

Cartesian Closed Categories

Corresponds to lambda calculus
Mapping to and from the lambda calculus
“A CCC is a category that has an exponent and a product, and is closed over both. The product is an abstract version of cartesian set product; the exponent is an abstraction of the idea of a function, with an “eval” arrow that does function evaluation.” [link]

Triumvirate

Role of category theory as one member of the triumvirate that includes type theory and mathematical logic
“Roughly speaking, a category may be thought of as a type theory shorn of its syntax” [link]