Mathematical Logic

Logic as Foundation, Propositional, First Order, Higher Order, Models, Temporal, Modal, Kripke

Mathematical logic permeates all of mathematics and programming.

Building on simple propositional logic, a host of richer logics can be constructed to target different needs. For example, First Order Logic is the deductive system used by set theory and is also the basis for Description Logic. Higher Order Logics have richer predicates though come with added complexity.

We need to look at what we can represent in logic and what valid claims we can make about such.

Logic can be qualified by time, by modality or other techniques. More advanced logics are based on this common foundation. It is useful to consider these logics as a set of building blocks which can be rearranged to suit specific needs. We are interested in building our own logics. We also want to use mathematical logic in real projects and explore how best to do this.

By the end of this course it will be clear that a good understanding of mathematical logic underpins a good understanding of all of mathematics and programming.

Target Audience

This course is aimed at mathematicians and modern developers who need a better grasp of how mathematical logic can be used in practice

Prerequisites

Good foundational mathematical education along with some programming experience, as we include exploring logic from a computational viewpoint. Attendees can select which programming language they wish to use, as all concepts will be developed from first principles.

Contents of One-Day Training Course

Logic Fundamentals

Defining and using a formal logic
Review of logic as branch of mathematics
Part of foundations
List of logics
Good understanding of logic helps with understanding everything above it

Propositional Logic

Conjunction
Disjunction
Negation
Conditional
Truth tables

Predicate (First Order) Logic

Builds on propositional logic
∀ means “for all”
∃ means “there exists”
Quantifiers
More advanced symbols
How first order logic is used in set theory

Higher Order Logic

Extra quantifiers
Predicates themselves having parameters
What variables range over (sets of sets)
Quantifying over sets
Additional semantics

Model Theory

Mathematical models
An interpretation gives meaning to symbols in a formal language
When is an interpretation a model?
Interpretation function
Domain

Structural Rules

* Weakening
* Contraction
* Exchange
* Associativity
Being selective - substructural

Modal Logic

Modality
Modal terms and their impact
In some way we wish to qualify or restrict a logic statement
Modal operators

Kripke Semantics

Usefulness
The Kripke model
Soundness of a modal logic
Kripe Frame

Temporal Logic

Logic that is in some way time-based
Now, future, past, until, while, always, …
Temporal operators
More fine-grained representation of things: how they are in the real world (beyond being simply always true)

Custom Logic

Why would we want to define our own logic?
Steps involved
How to create and use

Project

Practical use of a variety of mathematical logic in a production setting
Considerations and observations